

Algebra 2H

6.3 Add, Subtract, and Multiply Polynomials

6.4 Factoring and Solving Polynomial Equations

Goal: Add, subtract, and multiply polynomials.

Factor and solve polynomial expressions.

HW - Section 6.3 #22-25, 29, 38, 44, 50
 Section 6.4 39, 40, 46, 47, 55, 56, 77-82
 Quiz 6.1-6.4 Thursday (end of class)

Section 6.3

Adding and Subtracting Polynomials

- Add and subtract like terms (same variable, same exponent)
- Remember to distribute subtraction to every term in the polynomial

Examples:

1. $(15m^3 + 8m) + (2m^2 - 6m + 9)$

$$15m^3 + 2m^2 + 2m + 9$$

$$(b^4 + 10b) - (4b^3 + 6b^2 - b + 5)$$

3.
$$\begin{array}{r} b^4 \qquad \qquad +10b \\ -(4b^3 + 6b^2 - b + 5) \\ \hline b^4 - 4b^3 - 6b^2 + 11b - 5 \end{array}$$

2.
$$(4z^2 + 3) - (z^3 - 5z + 4)$$

$$-z^3 + 5z + 3$$

4.
$$(\sqrt{2}d^2 - 6d + 1) + (2\sqrt{2}d^2 + d - 8)$$

$$3\sqrt{2}d^2 - 5d - 7$$

Multiplying Polynomials

- When multiplying two terms with the same base, ADD exponents

Examples:

1. $8y^3(2y^4 - 5y)$

$$16y^7 - 40y^4$$

2. $(h-3)(h^2 + 2h - 8)$

$$h^3 + 2h^2 - 8h - 3h^2 - 6h + 24$$

$$h^3 - h^2 - 14h + 24$$

3. $(4c^3 + 5)(4c^3 - 5)$

$$16c^6 - 20c^3 + 20c^3 - 25$$

$$16c^6 - 25$$

factor as a quadratic.

4. $(2c+3)^3 \rightarrow \neq 2^3c^3 + 3^3 \neq 8c^3 + 27$

$$(2c+3)(2c+3)(2c+3)$$

$$(4c^2 + 6c + 6c + 9) \downarrow$$

$$(4c^2 + 12c + 9)(2c+3)$$

$$8c^3 + 12c^2 + 24c^2 + 36c + 18c + 27$$

$$8c^3 + 36c^2 + 54c + 27$$

If $f(x) = 2x^2$ and $g(x) = x^3 - 5$, find:

- A. $(f+g)(x)$
 $f(x)+g(x)$
- B. $(f-g)(x)$
 $f(x)-g(x)$
- C. $fg(x)$
 $f(x)g(x)$
- D. $f \circ g(x)$
 $f(g(x))$
- E. $g(f(x))$
 $g(f(x))$

A few more problems:

For 1-4, simplify each expression. Assume all variables represent positive integers.

1. $3x^b(5x^{2b} + 4x^{3b-1})$

2. $(6x^m - 5)(2x^{2m} - 3)$

3. Find the product $(a+b+c)^2$

4. Use your results from ~~#4~~^{#3} to find: $(x^2 + 3x - 2)^2$

Section 6.4 Factoring and Solving Polynomial Equations

Factoring

- Look for a common factor
 - Two terms? Try factoring as a difference of squares or as a sum or difference of cubes
 - Three terms? Try factoring as a square of a binomial or test the factors of the terms
 - More than three terms? Try factoring by grouping
 - Factor completely. Make sure each factor is prime
- like a quadratic ex (4c)

$$25c^2 - 4$$

$$\uparrow (5c+2)(5c-2)$$

Sum and Difference of Cubes

$$* a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$* a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Examples: Factor.

A. $27y^3 + 8$

$$(3y)^3 + (2)^3$$

$$\rightarrow (3y+2) \left((3y)^2 - (3y)(2) + (2)^2 \right)$$

$$\boxed{(3y+2)(9y^2 - 6y + 4)}$$

B. $216d^3 - 125$

$$(6d)^3 - (5)^3$$

$$\boxed{(6d-5)(36d^2 + 30d + 25)}$$

$$(6d)^2 + (6d \cdot 5) + (5)^2$$

$$2x^2y - 8y$$

Factoring by Grouping - Check for a common binomial factor or for a difference of squares

Examples: Factor.

A. $2x^3 + 6x^2 - 8x - 24$

$$2x^2(x+3) - 8(x+3)$$

* the binomials have to be the same. if not !!

$$(x+3)(2x^2 - 8)$$

$$2(x+3)(x^2 - 4)$$

$$\boxed{2(x+3)(x+2)(x-2)}$$

B. $y^2 - 9a^2 + 12y + 36$

$$y^2 + 12y - 9a^2 + 36$$

$$y(y+12) - 9(a^2 - 4)$$

$$\boxed{y(y+12) - 9(a+2)(a-2)}$$

C. $d^6 - 26d^3 - 27$ factor this like a quadratic.

$$(d^3 - 27)(d^3 + 1)$$

ex. $d^2 - 26d - 27$.

$$\left(\begin{matrix} d^3 - 3^3 \\ (a^3 - b^3) \end{matrix} \right) \left(\begin{matrix} d^3 + 1^3 \\ (a^3 + b^3) \end{matrix} \right)$$

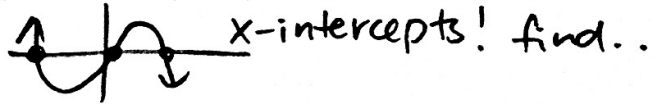
$$(d-27)(d+1)$$

$$\boxed{(d-3)(d^2 + 3d + 9)(d+1)(d^2 - d + 1)}$$

$$\rightarrow (a-b)(a^2 + ab + b^2)$$

Solving by Factoring

Set the equation equal to zero. Factor the polynomial completely. The solutions are the values that make each factor equal to zero.



Examples: Solve by factoring

A. $y^2 + 2y - 63 = 0$

$$(y+9)(y-7) = 0$$

$$\boxed{y = 7, -9}$$

B. $2x^2 = 4x$

$$2x^2 - 4x = 0$$

$$\boxed{2x(x-2) = 0}$$

$$\boxed{x = 0, 2}$$

C. $x^3 + 4x^2 - 9x - 36 = 0$ grouping

$$\underline{x^3 + 4x^2 - 9x - 36 = 0}$$

$$x^2(x+4) - 9(x+4) = 0$$

$$(x^2 - 9)(x+4) = 0$$

$$\boxed{(x+3)(x-3)(x+4) = 0}$$

D. $x^3 - 5x^2 - x + 5 = 0$

$$x^2(x-5) - 1(x-5) = 0$$

$$(x^2 - 1)(x-5) = 0$$

$$\boxed{(x+1)(x-1)(x-5) = 0}$$

$$\boxed{x = \pm 1, 5}$$

E. $50c^3 + 240c^2 - 50c = 0$ $\boxed{x = \pm 3, -4}$

$$\& 10c(5c^2 + 24c - 5) = 0$$

$$\boxed{10c(c+5)(5c-1) = 0}$$

$$\boxed{c = 0, -5, 1/5}$$

F. $8x^4 + 10x^2 - 3 = 0$

$$2 \cdot 4 \quad \underline{12} \quad 3, -1$$

$$2x^2 = -3$$

$$x^2 = -3/2$$

$$x = \pm i\sqrt{6}/2$$

$$(2x^2 + 3)(4x^2 - 1) = 0$$

$$\boxed{(2x^2 + 3)(2x+1)(2x-1) = 0}$$

$$\boxed{x = \pm 1/2, \pm i\sqrt{6}/2}$$

H. $4g^3 - g^2 - 4g + 1 = 0$

$$\underline{4g^3 - g^2 - 4g + 1 = 0}$$

$$g^2(4g-1) - 1(4g-1) = 0$$

$$(g^2 - 1)(4g-1) = 0$$

$$(g+1)(g-1)(4g-1) = 0$$

$$\boxed{g = \pm 1, 1/4}$$

G. $k^6 - 4k^2 = 0$

$$k^2(k^4 - 4) = 0$$

$$k^2(k^2 + 2)(k^2 - 2) = 0$$

$$\boxed{k = 0, \pm\sqrt{2}, \pm i\sqrt{2}}$$

I. $10u^4 - 8u^3 + 25u^2 - 20u = 0$

$$2u^3(5u-4) + 5u(5u-4) = 0$$

$$(2u^3 + 5u)(5u-4) = 0$$

$$u(2u^2 + 5)(5u-4) = 0$$

$$\boxed{u = 4/5, 0, \pm i\sqrt{10}/2}$$

$$2u^2 + 5 = 0$$

$$2u^2 = -5$$

$$u^2 = -5/2$$