ALGEBRA 42
REVIEW FOR CHAPTER 5

NAME Key

Evaluate the following by synthetic substitution:

1. $f(x) = x^3 - 4x^2 - 5x - 1$ when x = 4

-21

2. $h(x) = 5x^4 - 7x^3 - 2x^2 + 12x - 18$ when x = -1

	5	-7	-2	12	-18
-1	5	-12	10	2	(-20)

Decide whether the functions below are polynomial functions. If the function is a polynomial function, state its degree, number of terms, and leading coefficient.

3. $f(x) = 13 - 2x$

Polynomial function: **YES** or NO (circle)

Degree: 1

Constant: 13

Leading coefficient: -2

Classify by degree: linear

Classify by # of terms: binomial

4. $h(x) = 9x^4 - 5x^{-2} + 4$

Polynomial function: YES or **NO** (circle)

Degree: ____

Constant: ____

Leading coefficient: ____

Classify by degree:

Classify by # of terms:

5. $r(x) = 6x^2 + \pi - 3x$

Polynomial function: **YES** or NO (circle)

Degree: 2

Constant: π

Leading coefficient: 6

Classify by degree: Quadratic

Classify by # of terms: trinomial

Use infinity notation to describe the end behavior of the polynomial function:

6. $f(x) = -4x^7 + 2x + 3$

As $x \to +\infty$, $f(x) \to -\infty$

As $x \to -\infty$, $f(x) \to +\infty$

7. $f(x) = 3x^6 + x - 1$

As $x \to +\infty$, $f(x) \to +\infty$

As $x \to -\infty$, $f(x) \to +\infty$

Perform the following operations and write your final answer in standard form:

8. $(x^2 - 6x - 9)(x + 3)$

$x^3 - 3x^2 + 27x - 27$

9. $(2x + 4)^3$

$8x^3 + 48x^2 + 96x + 64$

10. Subtract $(5x^2 - 9x - 1)$ *from* $(6x^2 - 7x + 2)$

$x^2 + 2x + 3$

11. $(x^3 - 6x^2 + 1)(2x^3 - 7x^2 - 1)$

$2x^6 - 19x^5 + 42x^4 + x^3 - x^2 - 1$

12. $(x-4)(x-3)(x-2)$

$x^3-9x^2+26x-24$

13. $(4a-5b)^2$

$16a^2-40ab+25b^2$

Find the greatest common factor of the following:

14. $4x^3-20x^2-30x$

$2x$

15. $-5ab^3+15a^2b^2$

$-5ab^2$

Factor the following completely:

16. x^3+2x^2-15x

$x(x^2+2x-15)$

$x(x-3)(x+5)$

17. $2y^5-18y^3$

$2y^3(y^2-9)$

$2y^3(y-3)(y+3)$

18. $4x^4-16x^3+16x^2$

$4x^2(x^2-4x+4)$

$4x^2(x-2)(x-2)$

19. x^4-7x^2-18

$(x^2-9)(x^2+2)$

$(x-3)(x+3)(x^2+2)$

20. $x^3-3x^2-16x+48$

$x^2(x-3)-16(x-3)$

$(x-4)(x+4)(x-3)$

21. $25x^6-36$

$(5x^3-6)(5x^3+6)$

Solve the following by factoring:

22. $x^3=5x^2$

$x^3-5x^2=0$

$x^2(x-5)=0$

$x=0, 5$

23. $x^3+7x^2-9x-63=0$

$x^2(x+7)-9(x+7)=0$

$(x^2-9)(x+7)=0$

$x=\pm 3, -7$

24. $16x^2-1=0$

$(4x-1)(4x+1)=0$

$x=\frac{1}{4}, -\frac{1}{4}$

25. $3x^5+15x=18x^3$

$3x^5-18x^3+15x=0$

$3x(x^4-6x^2+5)=0$

$3x(x^2-5)(x^2-1)=0$

$x=0, \pm\sqrt{5}, \pm 1$

26. $y^5-3y^4-16y+48=0$

$y^4(y-3)-16(y-3)=0$

$(y^4-16)(y-3)=0$

$(y^2-4)(y^2+4)(y-3)=0$

$(y-2)(y+2)(y^2+4)(y-3)=0$

$y=\pm 2, \pm 2i, 3$

27. $10x^3-3x^2-4x=0$

$x(10x^2-3x-4)=0$

x

Applications: (you may use a calculator)

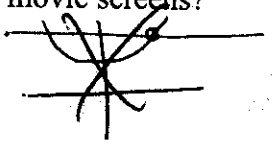
28. From 1987 to 2003, the number of indoor movie screens M in the United States can be modeled by the equation: $M=-11.0t^3+267t^2-592t+21600$ where t is the number of years since 1987.

a) How many indoor movie screens were there in 1998? 32,754 screens

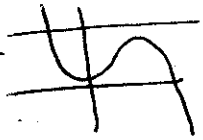
2nd Trace → 11 for x.

b) When were there 50000 movie screens?

$y_2=50000$



2nd Trace → Intersect



Never

29. Divide $5x^4 - 2x^2 + 5$ by $x^2 + 4$

$5x^2 - 22$ R 93

$x^2+0x+4 \overline{)\,5x^4+0x^3-2x^2+0x+5}$

$-(5x^4+0x^3+20x^2)$

$-22x^2+0x+5$

$-(-22x^2+0x-88)$

93

$$5x^2 - 22 + \frac{93}{x^2+4}$$

30. $(3x^5 - x^2 + 4x - 6) \div (2x - 3)$ $x = \frac{3}{2}$ is a zero

skip

	3	0	−1	4	−6
		$\frac{9}{2}$	$\frac{27}{4}$	$\frac{69}{8}$	$\frac{303}{16}$
$\frac{3}{2}$	3	$\frac{9}{2}$	$\frac{23}{4}$	$\frac{101}{8}$	$\frac{207}{16}$

$$3x^4 + \frac{9}{2}x^3 + \frac{23}{4}x^2 + \frac{101}{8}x + \frac{207}{16}$$

31. Use synthetic division to divide $(3x^4 - x^3 - 8x - 6)$ *by* $(x + 2)$

	3	−1	0	−8	−6
−2	3	−7	14	−36	66

$$3x^3 - 7x^2 + 14x - 36 + \frac{66}{x+2}$$

32. If one of the factors of $f(x) = 3x^3 - 4x^2 - 28x - 16$ is $x + 2$, what are the others?

	3	−4	−28	−16
−2	3	−10	−8	0

$3x^2 - 10x - 8$

$(3x + 2)(x - 4)$

33. One of the zeros of $f(x) = x^3 - 2x^2 - 23x + 60$ is $x = 3$. What are the other zeros?

	1	−2	−23	60
3	1	1	−20	0

$x^2 - x - 20 = (x - 5)(x + 4)$

$x = 5, -4, 3$

Find the zeros of each following. Indicate double zeros, you may use your calculator to narrow down the possible zeros. Watch out for the ones that are factorable!

34 $f(x) = x^3 - 3x^2 - 6x + 8$

	1	−3	−6	8
−2	1	−5	4	0

$x^2 - 5x + 4 = (x - 4)(x - 1)$

$x = 4, 1, -2$

35 $g(x) = x^3 + 4x^2 - x - 4$

$x^2(x+4) - 1(x+4) = 0$

$(x^2 - 1)(x + 4) = 0$

$x = \pm 1, -4$

36 $h(x) = x^3 + 4x^2 + x - 6$

	1	4	1	−6
1	1	5	6	0

$x^2 + 5x + 6 = (x + 2)(x + 3)$

$x = -2, -3, 1$

37 $g(x) = x^3 + 5x^2 - x - 5$

$x^2(x+5) - 1(x+5)$

$(x^2 - 1)(x + 5) = 0$

$x = \pm 1; -5$

38 $h(x) = 8x^3 - 6x^2 - 23x + 6$

$-\frac{3}{2}, \frac{1}{4}, 2$

39 $g(x) = 2x^4 + x^3 - x^2 - x - 1$

$-1, 1, \frac{-1 \pm i\sqrt{7}}{4}$

40 $h(x) = 2x^4 + 5x^3 - 5x^2 - 5x + 3$

$-3, -1, \frac{1}{2}, 1$

41 $f(x) = 2x^4 + 3x^3 - 6x^2 - 6x + 4$

$-2, \frac{1}{2}, \pm\sqrt{2}$

42 $g(x) = x^3 + 5x^2 + x + 5$

$-5, \pm i$

43 $g(x) = x^4 - 9x^3 + 23x^2 - 81x + 126$

$2, 7, \pm 3i$

44 $f(x) = x^3 - x^2 - 11x + 3$

$-3, 2 \pm \sqrt{3}$

45 $h(x) = 2x^4 + x^3 + x^2 + x - 1$

$-1, \frac{1}{2}, \pm i$

Use a graphing calculator to graph the function. Identify the *x*-intercepts and points where local maximums or local minimums occur.

46 $f(x) = 3x^3 - 9x + 1$

X-ints: $-1.79, .11, 1.67$

max: $(-1, 7)$ min: $(1, -5)$

47 $h(x) = -\frac{1}{3}x^2 + x - \frac{2}{3}$

X-ints: $-2, 1$

max $(1, 0)$

min $(-1, -\frac{4}{3})$

48 $g(x) = -\frac{1}{4}x^4 + 2x^2$

X-ints: $-2.83, 0, 2.83$

max: $(-2, 4), (2, 4)$

min $(0, 0)$

49 $f(x) = x^5 - 6x^3 + 9x$

X-ints: $-1.73, 0, 1.73$

max: $(-1.73, 0), (.77, 4.46)$

min: $(-.77, -4.46), (1.73, 0)$

State the maximum number of turns in the graph of the function.

50 $f(x) = x^4 + 2x^2 + 4$

3

51 $f(x) = -3x^3 + x^2 - x + 5$

2

52 $f(x) = 2x^6 + 1$

5

List the possible rational zeros of the function using the rational zero theorem.

53 $f(x) = x^4 - 6x^3 + 8x^2 - 21$

$\pm 1, \pm 3, \pm 7, \pm 21$

54 $h(x) = 2x^3 + 7x^2 - 7x + 30$

Constant: $1, 2, 3, 5, 10, 15, 30$

lead: $1, 2$

Possible: $\pm 1, \pm\frac{1}{2}, \pm 2, \pm 3, \pm\frac{3}{2}, \pm 5, \pm\frac{5}{2}, \pm 10, \pm 15, \pm\frac{15}{2}, \pm 30$

Write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1, and the given zeros.

55 $-7, -4$

$f(x) = x^2 + 11x + 28$

56 $1, 2, 5$

$f(x) = x^3 - 8x^2 + 17x - 10$

57 $8, 2 + i$

$f(x) = x^3 - 12x^2 + 37x - 40$

58 $-4, 2\sqrt{3}$

$x^3 + 4x^2 - 12x - 48$

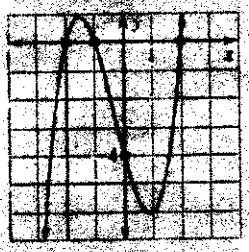
59. Graph accurately

$f(x) = \frac{1}{4}(x-2)^2(x-3)(x+1)$

Zeros: 2 (double)
3
-1

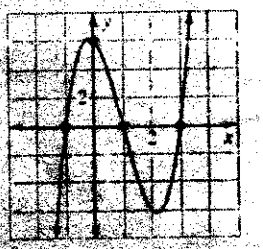
Write the cubic function whose graph is shown.

60



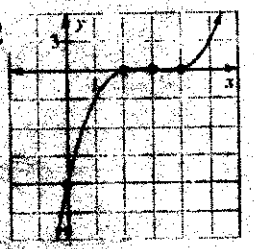
$f(x) = (x+2)(x+1)(x-2)$

61



$f(x) = 2(x+1)(x-1)(x-3)$

62



$f(x) = \frac{1}{2}(x-2)(x-3)(x-4)$

63 Which is *not* a possible rational zero of the function $f(x) = x^4 + 3x^3 - 7x^2 + 9x - 30$?

(A) -5 (B) -3 (C) 1
(D) 3 (E) 4