Algebra 2H
Semester Review

The final exam counts for 20% of your semester grade.
The final exam covers Chapters 6 – 10.

Chapter 6 – Polynomials and Polynomial Functions
- Add, subtract, multiply, and divide (synthetic and long) polynomials
- Solve polynomial functions (including factoring)
- Graph polynomials function

Chapter 7 – Rational Exponents and Radical Expressions
- Simplify expressions with radicals or rational exponents
- Perform function operations and compositions of functions
- Find inverses of functions
- Verify that two functions are inverses
- Graph square root and cube root functions
- Solve radical equations

Chapter 8 – Exponential and Logarithmic Functions
- Determine whether a function represents exponential growth or decay
- Simplify functions containing e
- Use exponential growth and decay models ($y = a(1 \pm r)^t$, $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$, $A = Pe^{rt}$)
- Graph exponential functions
- Evaluate logarithms
- Graph logarithmic functions
- Use properties of exponents to expand/condense expressions
- Solve exponential and logarithmic equations
- Write an exponential equation through two points

Chapter 9 – Rational Functions
- Graph rational functions and identify asymptotes
- Add, subtract, multiply, and divide rational expressions (including complex fractions)
- Solve rational equations

Chapter 10 – Conic Sections
- Find the distance between two points
- Write an equation of a graph of a conic section
- Graph conic sections
- Rewrite equations of conic sections by completing the square
- Write an equation of a conic section given some constraints (center, directrix, length of major/minor axis, etc)

General Stuff (pertains to radical, exponential, logarithmic, and rational functions)
Determine domain and range of functions
Graph functions and their transformations
Find inverses of functions

Simplify completely. There should be no negative exponents or decimals!

1. $-2x^3\left(-3x^2\right)^3$	2. $\dfrac{3xy^{-1}}{2xy}\div\dfrac{4x^{-1}}{3y^5}$	3. $\dfrac{4^{\frac{5}{2}}}{4^2}$
4. $\dfrac{\left(2r^{-1}s^2t^0\right)^{-2}}{2rs}$	5. $4\sqrt[3]{2x^5}-x\sqrt[3]{16x^2}$	6. $\left(\dfrac{8}{27}\right)^{\frac{1}{3}}\bullet\left(\dfrac{8}{27}\right)^{\frac{-2}{3}}$
7. $\sqrt[5]{\dfrac{x^{10}}{y^{12}}}$	8. $\log_6 36$	9. $\log 0.0001$
10. $\ln e^4$	11. $\log_4 1$	12. $\log_2(16)$
13. $\log\left(\sqrt[3]{10}\right)$	14. $\log_2(-8)$	15. $3\ln(y)-2\ln(x)$
16. $\dfrac{1}{2}\log_5 144$	17. $\log_7 10-4\log_7 5$	18. $\log_2 8+\log_2\dfrac{1}{2}-\log_2 1$
19. $\dfrac{2x^2+4x}{x^2-4}\div\dfrac{x^2-3x+2}{3x-6}$	20. $\dfrac{5x^2y}{4y^3}\bullet\dfrac{12x^2y^2}{30x^3}$	21. $\dfrac{5x+4}{x^2-64}+\dfrac{3}{x-8}$

22. $\dfrac{x+4}{x^2+2x+1}+\dfrac{x}{x^2-1}-\dfrac{2}{x-1}$	23. $\dfrac{\dfrac{2}{x}+\dfrac{3}{x-1}}{\dfrac{1}{2x-2}-\dfrac{1}{2}}$	24. $\dfrac{x^2+4x+3}{x^2+5x+6}\bullet\dfrac{x^2-3x-10}{x^2+x}$
25. $(x^2-6x-9)(x+3)$	26. $(2x+4)^3$	27. Subtract $(5x^2-9x-1)\;\;from\;\;(6x^2-7x+2)$

28. $(2x^3-11x^2+3x+36)\div(2x^2-5x-12)$	30. $(3x^3+17x^2+21x-11)\div(x+3)$

Solve each equation completely. Leave exact, simplified answers when possible and check for extraneous solutions!

31. $x^3=5x^2$	32. $x^3+7x^2-9x-63=0$	33. $3x^5+15x=18x^3$
34. $16x^2-1=0$		

35. $2(x-3)^4 = 32$	36. $\frac{1}{4}(x)^3 = 2$	37. $(x+7)^2 = 24$
38. $60 - \frac{1}{20}(x+75)^{\frac{3}{2}} = 10$	39. $x+1 = \sqrt{19-x}$	40. $\sqrt{2x+3} - \sqrt{x+1} = 1$
41. $x^3 = 64$	42. $2x^6 = 1458$	43. $\sqrt[5]{4x-8} = 2$
44. $10^{2x} = 57$	45. $e^{\frac{4x}{3}} = 18$	46. $3(0.8)^{2a} - 5 = 19$
47. $\log(3x+2) + \log(x-1) = 1$	48. $\log_2(2x) = 1 - \log_2(x)$	49. $\log_9(4x+1) - \log_9(2x-7) = \frac{1}{2}$

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50. $\log_5(a+3) - \log_5(a-1) = 1$	51. $\log_2 x = -4$	52. $6\ln\frac{3x}{2} = 5$
53. $\log_3(1-8x) = 2$	54. $7\ln 4x = 2$	55. $5 \bullet 3^x = 4 \bullet 2^x$
56. $\dfrac{2}{x-2} = \dfrac{4}{x+2}$	57. $\dfrac{-x+1}{x-1} + 2 = \dfrac{1}{x}$	58. $\dfrac{x}{2x-1} - \dfrac{2}{2x+1} = \dfrac{x^2+20}{4x^2-1}$
59. $\dfrac{5}{x-1} + \dfrac{x+2}{x^2-5x+4} = 3$	60. $\dfrac{7}{x-1} - 5 = \dfrac{6}{x^2-1}$	61. $\dfrac{3}{x-7} + \dfrac{9}{x^2-5x-14} = \dfrac{x+1}{x+2}$

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62. Find all real and imaginary zeros of $f(x) = 2x^3 - 5x^2 + 7x - 4$. (Be sure to note whether any of them are double or triple zeros.)

63. Find all real and imaginary zeros of $g(x) = x^4 - 2x^3 - 6x^2 + 14x - 7$. (Be sure to note whether any of them are double or triple zeros.)

64. Write a polynomial function in standard form of least degree that has rational coefficients, a leading coefficient of 1, and zeros at 3, $3 - i$?

Graph each function. Identify the domain and range.
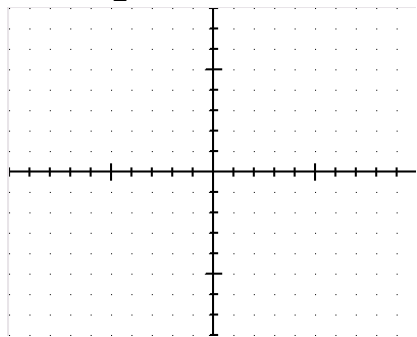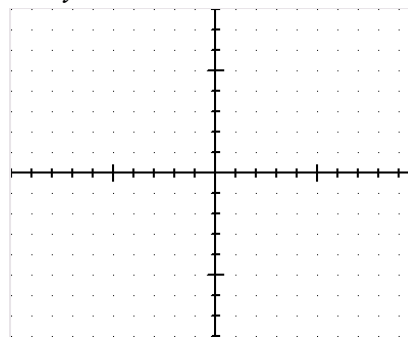
65. $y = \dfrac{1}{2}\sqrt[3]{x-3} + 1$	66. $y = -2\sqrt{x+4} - 1$
67. $f(x) = \log_2(x) - 4$	68. $f(x) = \dfrac{1}{2} \cdot 4^{x+2} - 1$

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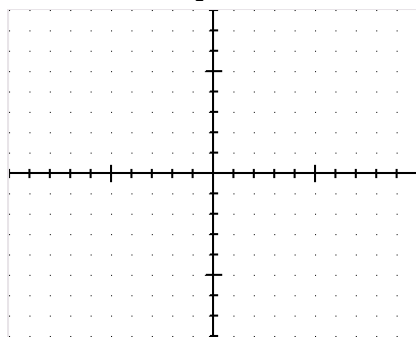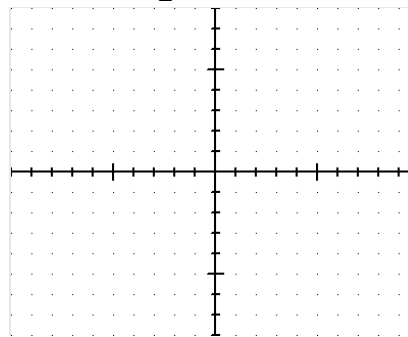
69. $f(x) = \log_3(x+6)$
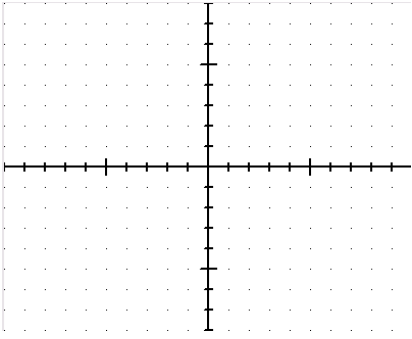
70. $f(x) = -2e^x + 5$
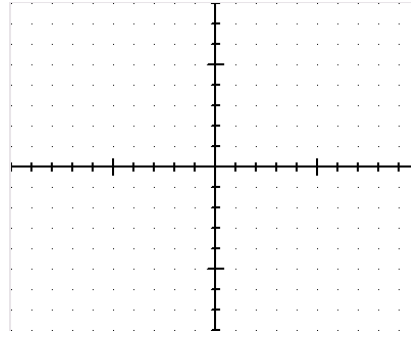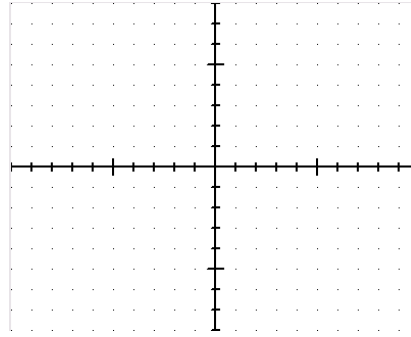
71. $f(x) = \ln(x+2) - 2$
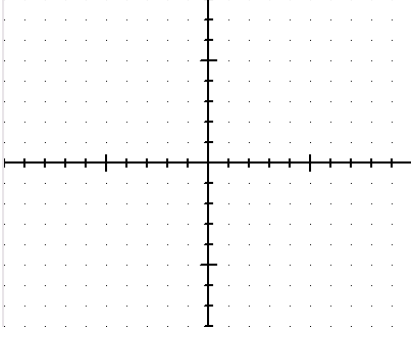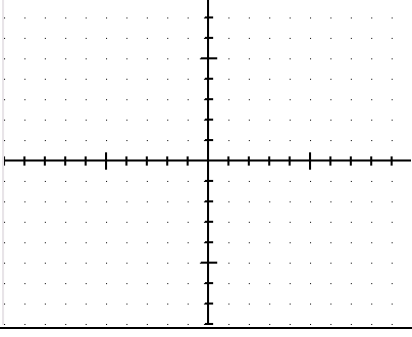
72. $f(x) = \dfrac{4}{x+3} - 2$
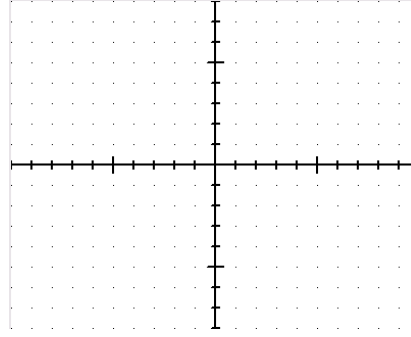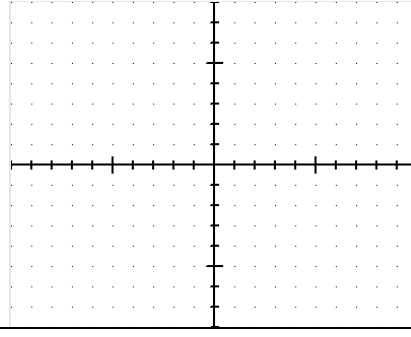
73. $f(x) = \dfrac{-12x}{6x+3}$
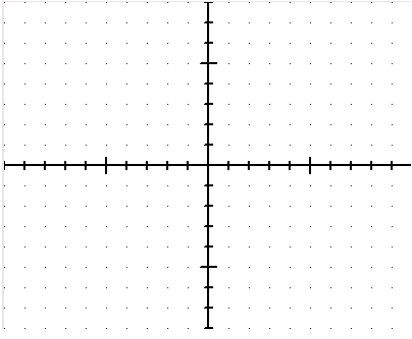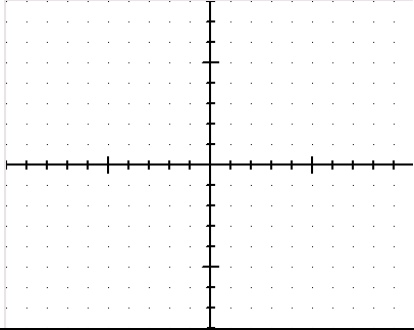
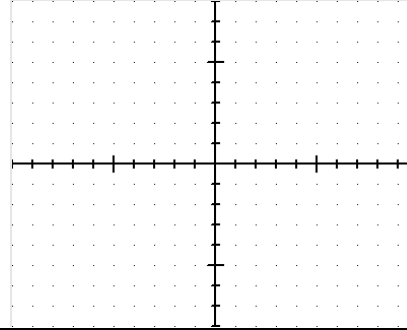74. $y = \dfrac{-3x^2}{(x+3)(x-2)}$
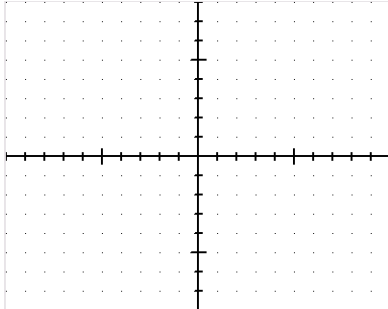
75. $-8(y+2) = (x-3)^2$

76. $-2x = (y-4)^2$
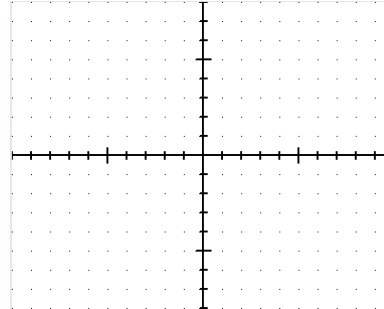
77. $\dfrac{(x-2)^2}{9} + \dfrac{(y+1)^2}{16} = 1$

78. $\dfrac{(x+3)^2}{25} - \dfrac{(y)^2}{36} = 1$
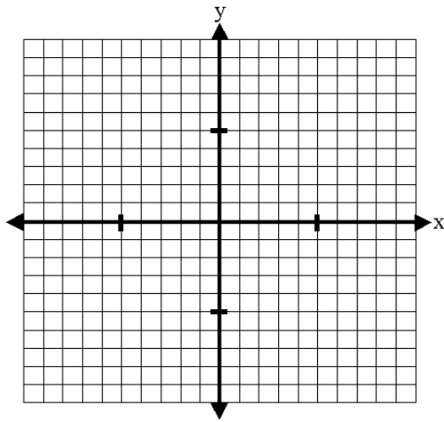
79. $x^2 + y^2 + 6x - 8y - 7 = 4$

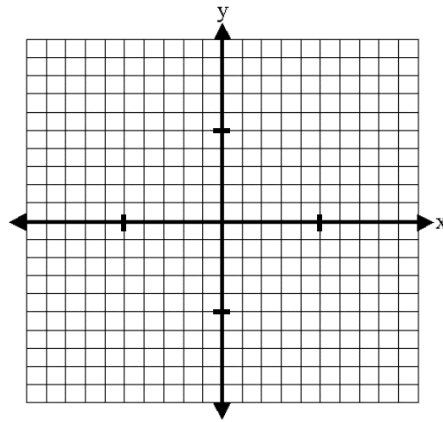80. $4x^2 + y^2 - 8x - 8 = 0$

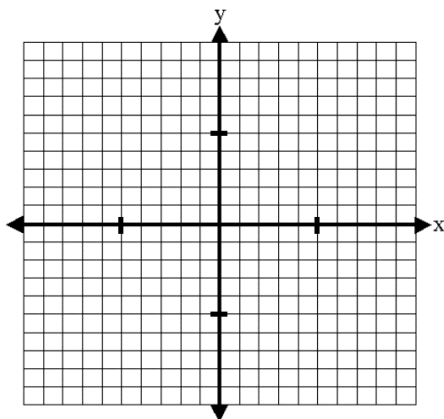81) Find all real zeros and graph each polynomial function.
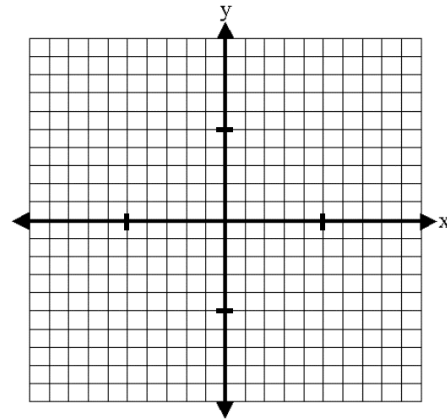
a) $y = x^2(x-1)(x+1)^2$

b) $f(x) = x^3 - 3x - 2$

c) $y = x^4 - 6x^2 + 8$

d) $f(x) = x^3 - 4x^2 - 3x + 12$

8

82. Verify that f and g are inverse functions using composition: $f(x)=2x^7$ and $g(x)=\sqrt[7]{\dfrac{x}{2}}$

83. Find the inverse of each function.

a) $f(x)=\dfrac{3}{2}x^3+4$

b) $f(x)=3x^8, x\geq 0$

c) $f(x)=\dfrac{4-5x}{2}$

d) $g(x)=3^x$

e) $h(x)=\dfrac{1}{2}\cdot 5^{x+1}$

f) $g(x)=-3x+4$

84. Find the horizontal and vertical asymptotes of the following functions.

a) $y=\dfrac{x+3}{x^3-4x}$

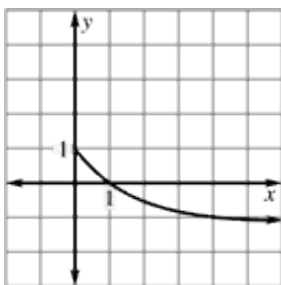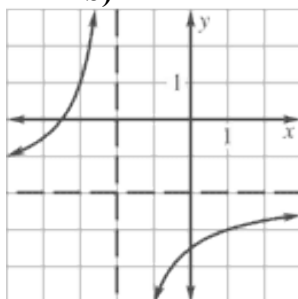b) $y=\dfrac{2x-1}{x^3+3x^2+2x}$

c) $g(x)=\dfrac{x+3}{x^2+x-6}$

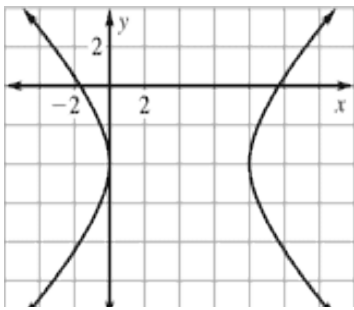d) $h(x)=\dfrac{2x^3-3x}{x^3-1}$

85. **Write an equation for each graph.**
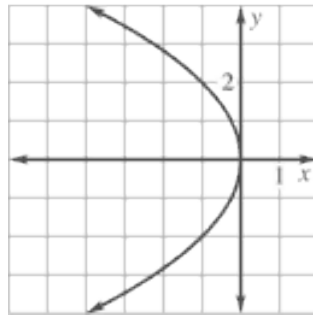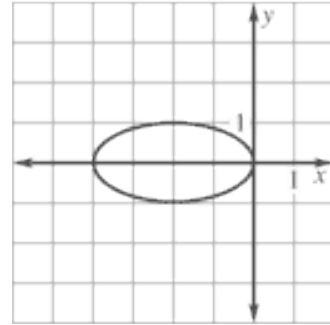a)   b)

9

c)

d)

e)

86. The projected worth (in millions of dollars) of a large company is modeled by the equation $y = 256(1.04)^x$. The variable x represents the number of years since 1997.

   What is the annual percent of growth?

   When will the company be worth 300 million dollars?

87. Write an exponential function to model the situation then estimate the value of the function after 5 years:

   A population of 490 animals decrease at an annual rate of 7%

88.   How much money must be deposited now in an account paying 8% annual interest, compounded quarterly, to have a balance of $1000 after 10 years?

89. Marion decided to invest $6000 at 5% interest compounded continuously. Find the value of his investment after 7 years.

90. There are initially 2000 bacteria in a culture. The number of bacteria, N, after t hours can be found using the formula $N = 2000(2)^t$.

What is the hourly percent of growth?

How long will it take the culture to grow to 60,000 bacteria?

91. Write an exponential function of the form $y = ab^x$ whose graph passes through the points (4 , 8) and (7 , 9).

You will need to know how to graph these functions without a calculator.

1. $f(x) = \sqrt[3]{x} - 1$ _____

2. $f(x) = \sqrt[3]{x-3} - 1$ _____

3. $f(x) = \sqrt{x+3} + 1$ _____

4. $f(x) = -\sqrt[3]{x} - 1$ _____

5. $f(x) = \sqrt{x} - 1$ _____

6. $f(x) = \sqrt{x} - 3$ _____

7. $f(x) = 4^{x-1}$ _____

8. $f(x) = 4^x - 1$ _____

9. $f(x) = \left(\frac{1}{2}\right)^{x+4}$ _____

10. $f(x) = \left(\frac{1}{2}\right)^x + 4$ _____

11. $f(x) = \ln\left(x - \frac{3}{2}\right)$ _____

12. $f(x) = -\frac{3}{2}\ln x$ _____

13. $f(x) = \dfrac{x+3}{x-2}$ _____

14. $f(x) = \dfrac{-8}{x^2 - 4}$ _____

15. $f(x) = \dfrac{3}{x+2}$ _____

16. $f(x) = \dfrac{x-3}{2x+4}$ _____

17. $\dfrac{(x-3)^2}{9} + \dfrac{(y+5)^2}{4} = 1$ _____

18. $(x-5)^2 + (y-3)^2 = 4$ _____

19. $\dfrac{(x+3)^2}{9} - \dfrac{(y-5)^2}{4} = 1$ _____

20. $(y-3)^2 - (x+5) = 0$ _____

ALGEBRA 2H
SEMESTER REVIEW KEY

1) $54x^9$

2) $\dfrac{9xy^3}{8}$

3) 2

4) $\dfrac{r}{8s^5}$

5) $2x\sqrt[3]{2x^2}$

6) $\dfrac{3}{2}$

7) $\dfrac{x^2}{y^2\sqrt[5]{y^2}}$

8) 2

9) -4

10) 4

11) 0

12) 4

13) $\dfrac{1}{3}$

14) not possible

15) $\ln\left(\dfrac{y^3}{x^2}\right)$

16) $\log_5 12$

17) $\log_7\left(\dfrac{2}{125}\right)$

18) 2

19) $\dfrac{6x}{(x-2)(x-1)}$

20) $\dfrac{x}{2}$

21) $\dfrac{4(2x+7)}{(x-8)(x+8)}$

22) $\dfrac{-6}{(x+1)(x+1)(x-1)}$

23) $\dfrac{2(5x-2)}{x(2-x)}$

24) $\dfrac{x-5}{x}$

25) $x^3-3x^2-9x+27$

26) $8x^3+48x^2+96x+64$

27) x^2+2x+3

28) $x-3$

30) $3x^2+6x+3-\dfrac{20}{x+3}$

31) x = 0, 5

32) $x = \pm 3, -7$

33) $x = 0,1,\pm\sqrt{5}$

34) $x = \pm\dfrac{1}{4}$

35) x = 5, 1

36) x = 2

37) $x = -7\pm 2\sqrt{6}$

38) x = 25

39) x = 3

40) x = 3, -1

41) x = 4

42) x = -3, 3

43) x = 10

44) x = .88

45) x = 2.17

46) n = -4.66

47) $x = \dfrac{1+\sqrt{145}}{6}$

48) $x = 1$

49) $x = 11$

50) $a = 2$

51) $x = \dfrac{1}{16}$

52) $x = 1.53$

53) $x = -1$

54) $x = .33$

55) $-.55$

56) $x = 6$

57) no solutions

58) $x = -3, 6$
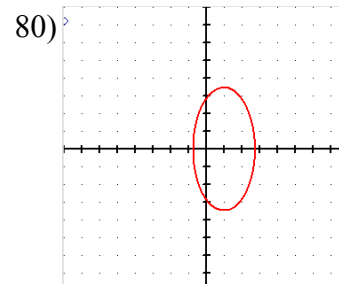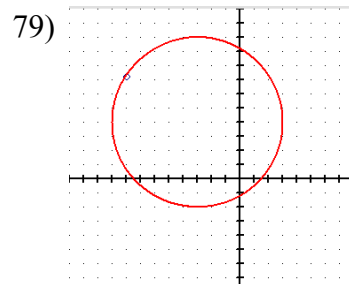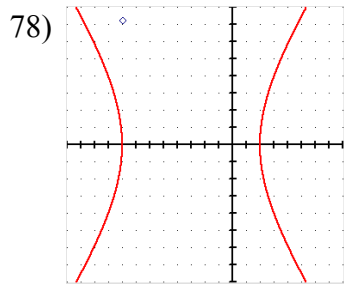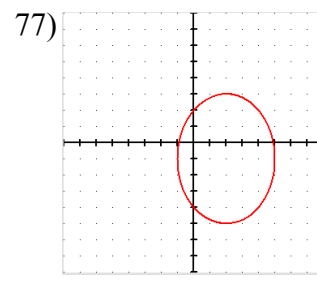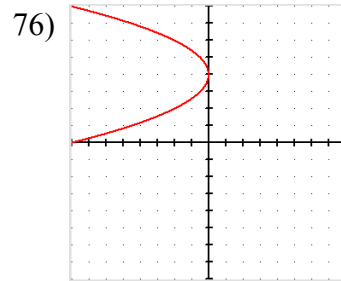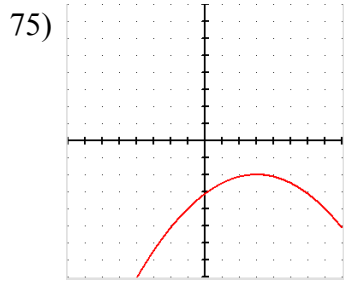
59) $x = 2, 5$

60) $x = -3/5, 2$

61) $x = 11$

62) $x = 1, \quad \dfrac{3 \pm i\sqrt{23}}{4}$

63) $x = 1$ (double), $\pm\sqrt{7}$

64) $f(x) = x^3 - 6x^2 + 19x - 30$

65) - 74) check answers with calculator!

75)

76)

77)

78)

79)

80)

81) Use your calculator to check your graphs.

82) $f(g(x)) = x$ and $g(f(x)) = x$

83) a. $f^{-1}(x) = \sqrt[3]{\dfrac{2x-8}{3}}$

b. $f^{-1}(x) = \sqrt[8]{\dfrac{x}{3}}$

c. $f^{-1}(x) = \dfrac{2x-4}{-5}$

d. $g^{-1}(x) = \log_3 x$

e. $h^{-1}(x) = \log_5(2x) - 1$

f. $g^{-1}(x) = \dfrac{x-4}{-3}$

84) a. $y = 0$, $x = -2, 0, 2$

b. $y = 0$, $x = -2, -1, 0$

c. $y = 0$, $x = -3, 2$

d. $y = 2$, $x = 1$

85) a) $y = -\sqrt{x} + 1$

b) $y = \dfrac{-3}{x+2} - 2$

c) $\dfrac{(x-4)^2}{4} - \dfrac{(y+4)^2}{4} = 1$

14

d) $y^2 = -4x$

e) $\dfrac{(x+2)^2}{4} + \dfrac{y^2}{1} = 1$

86) 4%, ~4.04 years

87) $y = 490(.93)^x$

88) $452.89

89) $8514.41

90) 100%, ~4.91 hours

91) $y = 6.8 \cdot 1.04^x$

Graphs 1-20 Sketch them!